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FITTING MARKOV RANDOM FIELD MODELS TO IMAGES

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Abstract

We are interested in fitting two-dimensional, Gaussian conditional Markov random field (CMRF) models to images. The given finite image is assumed to be represented on a finite lattice of specific structure, obeying a CMRF model driven by correlated noise. The stochastic model is characterized by a set of unknown parameters. We describe two sets of experimental results. First, by assigning values to parameters in the stationary range, two-dimensional patterns are generated. It appears that quite a variety of patterns can be generated. Next, we consider the problem of estimating the unknown parameters of a given model for an image, and suggest a consistent estimation scheme. We also implement a decision rule to choose an appropriate CMRF model from a class of such competing models. The usefulness of the estimation scheme and the decision rule to choose an appropriate model is illustrated by application to synthetic patterns. Unilateral approximations to CMRF models are also discussed.

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We are interested in fitting two-dimensional, Gaussian conditional Markov random field (CMRF) models to images. The given finite image is assumed to be represented on a finite lattice of specific structure, obeying a CMRF model driven by correlated noise. The stochastic model is characterized by a set of unknown parameters. We describe two sets of experimental results. First, by assigning values to parameters in the stationary range, two-dimensional patterns are generated. It appears that quite a variety of patterns can be generated. Next, we consider the problem of estimating the unknown parameters

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of a given model for an image, and suggest a consistent estimation scheme. We also implement a decision rule to choose an appropriate CMRF model from a class of such competing models. The usefulness of the estimation scheme and the decision rule to choose an appropriate model is illustrated by application to synthetic patterns. Unilateral approximations to CMRF modles are also discussed.

1. Introduction

Conditional Markov random field (CMRF) models have many applications in image processing and analysis; for instance, they can be used for the design of image restoration algorithms [1-4], for characterization of textures [4-7], and for image coding [4] applications. Apart from their applications in image processing, CMRF models are an active research topic in the statistical literature [8-9].

Typically, an image is represented by two-dimensional scalar data, gray level variations defined over a square grid. One of the important characteristics of such data is the statistical dependence of the gray levels within a neighbor set. For example, $y(i,j)$, the scalar gray level at position (i,j), might be statistically dependent on the gray levels over a neighbor set that includes $\{(i-1,j),(i+1,j),(i,j-1),(i,j+1)\}$. Due to the lack of a natural preferred direction in the grid as compared with the discrete time interpretation of one-dimensional Markov processes, the classical definitions of "past" and "future" are not generalizable to Markov random fields.

Suppose that we assume that $y(i,j)$ is statistically dependent on the nearest neighbors along the east, west, north and south directions. Then the Markov property in the corresponding CMRF model is induced by requiring that the conditional probability distribution of $y(i,j)$, given the values at all other sites, should depend only upon the values at the four sites nearest to (i,j), namely, $y(i-1,j)$, $y(i+1,j)$, $y(i,j-1)$ and $y(i,j+1)$. Wider classes of CMRF models can be considered by including dependence upon the values at more remote sites. For instance, corresponding to the first order Markov model mentioned above, we can have a second order CMRF model that includes dependence on the eight nearest neighbors, and so on. In this paper, we are primarily interested in Gaussian Markov fields, i.e., the conditional distribution of $y(i,j)$ given all other lattice observations is normally distributed with a mean that is a linear function of the observations at neighboring sites and with constant variance (say) ν. In these models, the observation at $y(i,j)$ is written as a linear weighted sum of the observations over the corresponding neighbor set and a <u>correlated</u> noise sequence, and is characterized by a set of coefficients and the variance of the noise driving the model.

Prior to the use of these models, two problems have to be tackled, namely, the estimation of the unknown parameters and the choice of an appropriate neighbor set for the given image. The first problem has received some attention in the literature. Besag [8] developed coding schemes for the estimation of parameters in CMRF models and maximum likelihood (ML) estimation schemes [9] for a first order isotropic CMRF model, by assuming a Gaussian structure for the noise. In the coding method the grid points (or sites) are divided into two sets (say A and B), such that the conditional distributions of gray levels associated with the members of set A given the observed gray levels at all other grid points are mutually independent. Once the forms of the conditional distributions are specified (except for a few unknown parameters), an expression for the conditional likelihood can be written, as the product of the conditional distributions over the members in set A. By maximizing this likelihood function, conditional maximum likelihood estimates of the unknown parameters can be obtained. One of the main disadvantages of this method is that the estimates thus obtained are not efficient [9] due to a partial utilization of the data. Though by complicated coding schemes [10], the utilization rates may be improved, the basic problem still remains. Also, for a particular CMRF model, more than one coding scheme can be realized, yielding several estimates likely to be highly dependent. Some arbitrary methods are used to combine these estimates. This coding approach has been used for the analysis of geographical data [8] and, very recently, in texture analysis [7] for the case of binomial variables.

Unlike the cases of one-dimensional time series models or two-dimensional unilateral models, deriving an expression for the likelihood of the observations poses some difficulties for CMRF models that include dependence on neighbors along all directions. This is due to the fact that the Jacobian of the transformation matrix from the noisy variates to the observations is difficult to evaluate. The problem of evaluating this determinant can be avoided by making assumptions about the representation of the underlying lattice. Specifically, by assuming representation on a toroidal lattice

[4-6,9], explicit expressions can be derived for the determinant term, as the transformation matrix possesses a block circulant structure whose eigenvalues can be written down explicitly. We use this representation and write explicit expressions for the likelihood of the observations. Since the likelihood function is nonquadratic in the parameters, the estimates are determined by using numerical optimization procedures such as Newton-Raphson, etc.

An alternative estimation scheme that yields consistent estimates for the parameters is given. These estimates are similar to the "least square" estimates in autoregressive time series models (we point out later the inappropriateness of using the term "least square" estimates in CMRF models). The estimation method does not involve computations by gradient methods, but involves inverting an $m \times m$ symmetric matrix for a CMRF model with dependence on 2m neighbors.

The second problem considered in fitting CMRF models is the choice of appropriate neighbors in images. From one dimensional time series analysis it is known that the use of an appropriate model leads to good results in forecasting and similar applications. This problem has been considered in the literature [4]; the decision rules developed are asymptotically consistent, transitive and parsimonious. They are based on the corresponding decision rules for discriminating between different autoregressive models [11]. We implement such a decision rule for choosing between different neighbor sets.

The usefulness of the estimation scheme and the decision rule for the choice of neighbors is demonstrated by application to synthetic patterns, the underlying true model of the synthetic patterns being known. This leads us to the problem of generating synthetic patterns. Computationally elegant solutions using torus representations for generating synthetic patterns obeying CMRF models have been developed in [4]. We use this approach to generate a number of two-dimensional Markov patterns. The patterns are quite varied and some of them possess repetitive periodic patterns. One of these synthetic patterns is used for illustrating the usefulness of the estimation scheme.

In a previous paper [12], we considered fitting another class of spatial autoregressive models, the so-called simultaneous autoregressive (SAR) models. Specifically, we suggested an approximate ML estimation scheme and implemented a decision rule for the choice of appropriate neighbor sets. The CMRF models considered in this paper are non-equivalent to the SAR models considered in [12], in that given a SAR model an equivalent CMRF model can always be found (usually with more parameters); however, the converse is not true. For instance, there is no equivalent SAR model for the first order CMRF model. The SAR models can be thought of as higher order Markov models. For instance, the conditional probability distribution function of a four-neighbor SAR model with dependence on the east, west, north and south neighbors is a function of the eight nearest neighbors and the second nearest neighbors on the east, west, north and south. Some of these points are illustrated later using synthetic patterns.

Another topic in this paper is unilateral approximation to CMRF models. Following Besag [8], we define the unilateral neighbor set of any site (i,j) in the positive quadrant to consist of those sites (k,ℓ) on the lattice which satisfy either (i) $\ell<j$ or (ii) $\ell=j$ and $k<i$. Such neighbor sets are of interest in Kalman filtering of noisy images [1,13]. We take a specific case of a first order CMRF model and discuss several unilateral approximations. The patterns corresponding to the first order CMRF model and its unilateral approximations look very similar in structure.

The organization of the paper is as follows: In Section 2, we consider the estimation problem and suggest a consistent estimation scheme. The implementation of the decision rule for the choice of appropriate neighbors is discussed in Section 3. In Section 4 experimental results are given.

2. Estimation of Parameter in CMRF Models

Assume that the observations $\{y(s), s\in\Omega\}$ have zero mean and obey the CMRF model in (2.1), the neighbor set of dependence being denoted by N_1:

$$y(s) = \sum_{(k,\ell)\in N_1} \theta_{k,\ell}[y(s + (k,\ell)) + y(s - (k,\ell))] + \sqrt{\nu}\, e(s) \quad , s\in \Omega \tag{2.1}$$

In (2.1), N_1 is a neighbor set such that if $(k,\ell)\in N_1$, then $(-k,-\ell) \notin N_1$, i.e., in our notation $N_1 = \{(0,1),(1,0)\}$ means that the pixel at (i,j) is statistically dependent on neighbors $\{(0,1),(0,-1),(1,0),(-1,0)\}$. The noise sequence $e(s)$ is correlated with zero mean and unit variance and $(\underset{\sim}{\theta},\nu)$ are unknown.

Let

$$\underset{\sim}{q}(s) = \text{col}[y(s + (k,\ell)) + y((s) + (-k,-\ell)), (k,\ell)\in N_1]$$

Then for a CMRF model we have

$$E(e(s)\underset{\sim}{q}(s)) = 0 \quad , \forall s \tag{2.2}$$

To ensure homogeneity, the coefficients $\{\theta_{k,\ell}, (k,\ell)\in N_1\}$, must obey

$$\mu'_{ij}(\underset{\sim}{\theta}) > 0 \tag{2.3}$$

where

$$\mu'_{ij}(\underset{\sim}{\theta}) = (1-2\underset{\sim}{\theta}^T \underset{\sim}{\phi}_{ij}) \tag{2.4}$$

and

$$\underset{\sim}{\phi}_{ij} = \text{col}[\cos \frac{2\pi}{M}((i-1)k + (j-i)\ell), (k,\ell)\in N_1] \tag{2.5}$$

For a finite image, and a CMRF in which neighbors in all direction are considered, some of the neighbors for the boundary sites are not defined. Several different assumptions can be made regarding the distribution of these pixels [4] [14]. In this paper we assume that the image is folded on a toroidal lattice such that, for all $(i,j)\in\Omega$,

$$y[(s) + (i_1,j_1)] = y[(s) + (i_1-1,j_1-1)\bmod M + 1] \quad (2.6)$$

This assumption ensures that all the relevant neighbors of any y(s) belonging to the finite image are well defined. Letting $\underset{\sim}{e}$ be the lexicographic ordered array of $\{e(s), s\ \Omega\}$, (2.1) can be rewritten as

$$H(\underset{\sim}{\theta})\underset{\sim}{y} = \sqrt{\nu}\ \underset{\sim}{e} \quad (2.7)$$

where

$$\underset{\sim}{\theta} = \mathrm{col}[\theta_{k,\ell}, (k,\ell)\in N_1]$$

$H(\underset{\sim}{\theta})$ is a block circulant matrix

$$H(\underset{\sim}{\theta}) = \begin{bmatrix} H_{1,1} & H_{1,2} & \cdots & & H_{1,M} \\ H_{1,M} & H_{1,1} & H_{1,2} & \cdots & H_{1,M-1} \\ \cdots & \cdots & \cdots & & \cdots \\ H_{1,2} & & & & H_{1,1} \end{bmatrix} \quad (2.8)$$

and each component matrix $H_{1,j}$ is circulant. For example, when $N_1 = \{(0,1),(1,0)\}$ we have

$$H_{1,1} = \text{circulant } (1,\theta_{0,1},0,\ldots,\theta_{0,-1})$$

$$H_{1,2} = \text{circulant } (\theta_{1,0},0,\ldots,0)$$

$$H_{1,M} = \text{circulant } (\theta_{-1,0},0,\ldots,0)$$

and $H_{1,j} = 0, \quad j \neq 1,2,\ldots,M$

Given an image, we are interested in fitting a CMRF model to it. This problem has received some attention in the literature. In [8], Besag developed coding schemes for estimation, and ML schemes were used in [9] for simple CMRF models. As mentioned before, the coding approach yields inefficient estimates due to a partial utilization of data. ML estimates can be derived by assuming some appropriate distributions for the noise sequence $\{e(s), s\in\Omega\}$. Assume, for instance, that $e(s)$ is distributed normally with zero mean and unit variance. Using this assumption, the log likelihood function $\ln p(\underset{\sim}{y}|\underset{\sim}{\theta},\nu)$ can be written as

$$\ln p(\underset{\sim}{y}|\underset{\sim}{\theta},\nu) = \ln \det H(\underset{\sim}{\theta}) - (M^2/2)\ln 2\pi\nu - \frac{1}{2\nu}\underset{\sim}{y}^T H(\underset{\sim}{\theta})\underset{\sim}{y} \quad (2.9)$$

From the theory of block circulant matrices,

$$\ln \det H(\underset{\sim}{\theta}) = \sum_{(i,j)\in\Omega} \ln \mu'_{ij}(\underset{\sim}{\theta}) \quad (2.10)$$

Using (2.4), (2.9) and (2.10) we have

$$\ln p(\underset{\sim}{y}|\underset{\sim}{\theta},\nu) = \sum_{(i,j)\in\Omega} \ln (1-2\underset{\sim}{\theta}^T\underset{\sim}{\phi}_{ij}) - (M^2/2)\ln 2\pi\nu - \frac{1}{2\nu}\underset{\sim}{y}^T H(\underset{\sim}{\theta})\underset{\sim}{y} \quad (2.11)$$

Since $H(\underset{\sim}{\theta})$ is a block-circulant matrix, it is diagonalized by a two-dimensional FFT, with eigenvalues $\mu'_{ij}(\underset{\sim}{\theta})$. Using this fact, we have

$$\underset{\sim}{y}^T H(\underset{\sim}{\theta})\underset{\sim}{y} = \sum_{\Omega} \| z(\lambda_{ij}) \| (1-2\underset{\sim}{\theta}^T\underset{\sim}{\phi}_{ij})$$

where $z(\lambda_{ij})$ are the Fourier transforms of $y(s)$, $s\in\Omega$ and

$\lambda_{ij} = (\frac{2\pi}{M}(i-1), \frac{2\pi}{M}(j-1))$. Substitution of (2.12) into (2.11) gives

$$\ln p(\underset{\sim}{y}|\underset{\sim}{\theta},\nu) = \sum_{\Omega} \ln (1-2\underset{\sim}{\theta}^T \underset{\sim}{\phi}_{ij}) - (M^2/2) \ln 2\pi\nu - \frac{1}{2\nu} \sum_{\Omega} \|z(\lambda_{ij})\|^2 (1-2\underset{\sim}{\theta}^T \underset{\sim}{\phi}_{ij}) \tag{2.13}$$

Note that the contribution of the exponent term of the likelihood function is linear in $\underset{\sim}{\theta}$, unlike the case of SAR models [12], where this term is quadratic. Consequently, the notion of least squares estimates does not extend to CMRF models. Numerical optimization procedures such as Newton-Raphson can be applied to obtain ML estimates. The ML estimates thus obtained are asymptotically consistent and efficient. However, the computation of ML estimates might become expensive due to the calculation of the gradients of the log likelihood function. An approximate ML estimation scheme for CMRF models, without substantial sacrifice in accuracy, is considered elsewhere [15]. To avoid these computational difficulties, we suggest the following estimation scheme: consider the estimates

$$\hat{\underset{\sim}{\theta}}_0 = [\sum_{\Omega} \underset{\sim}{q}(s)\underset{\sim}{q}^T(s)] \; (\sum_{\Omega} \underset{\sim}{q}(s)y(s)) \tag{2.14}$$

and

$$\hat{\nu}_0 = \frac{1}{M^2} \sum_{\Omega} (y(s) - \underset{\sim}{\theta}_0^T \underset{\sim}{z}(s))^2 \tag{2.15}$$

The easily computable estimate $\hat{\underset{\sim}{\theta}}_0$ is asymptotically consistent, as we will now prove.

<u>Theorem 1</u>: Let $y(s), s \in \Omega$ be the set of observations obeying

the CMRF model (2.1). Then the estimate $\hat{\underset{\sim}{\theta}}_0$ is consistent.

<u>Proof</u>: From (2.1) and (2.14),

$$\hat{\underset{\sim}{\theta}}_0 = [\sum_\Omega \underset{\sim}{q}(s)\underset{\sim}{q}^T(s)]^{-1} (\sum_\Omega \underset{\sim}{q}(s)(\underset{\sim}{\theta}^T\underset{\sim}{q}(s) + \sqrt{\nu}e(s))$$

$$= \underset{\sim}{\theta} + [\sum_\Omega \underset{\sim}{q}(s)\underset{\sim}{q}^T(s)]^{-1}\sqrt{\nu}e(s) \qquad (2.16)$$

Equivalently,

$$[\sum_\Omega \underset{\sim}{q}(s)\underset{\sim}{q}^T(s)] (\hat{\underset{\sim}{\theta}}_0 - \underset{\sim}{\theta}) = \sqrt{\nu}e(s) \qquad (2.17)$$

Since the matrix $\sum_\Omega \underset{\sim}{q}(s)\underset{\sim}{q}^T(s)$ is positive definite, and $E(\underset{\sim}{q}(s)e(s)) = 0$ with probability 1, the assertion of the theorem follows.

However, the estimate $\hat{\underset{\sim}{\theta}}_0$ is not efficient since it is not equal to the ML estimate obtained by maximizing (2.13). In this paper we use the estimate in (2.14) and (2.15) for simulation studies.

3. Model Selection

We briefly discuss the decision rules for the choice of appropriate CMRF models. The importance of appropriate model selection was illustrated in the case of SAR models [12],[15], using synthetic patterns. Specifically, it was shown that the quality of reconstruction varied considerably depending upon the underlying model. We can expect a similar situation with respect to CMRF models. The problem of choosing an appropriate CMRF model becomes difficult due to the rich variety of possible model structures. Suppose we have an original image, say a 64 x 64 window from one of the Brodatz textures, and we fit different CMRF models to the texture. It can be argued that by visual inspection of the reconstructed patterns corresponding to the different fitted models, a decision can be made regarding the appropriate model. However, there are several objections to this procedure. First, the decision rule is subjective and no quantitative measure of possible error in the decision is given. More significantly, the reconstructed patterns corresponding to an original RF model and another RF model which includes the original model and some extra neighbors look very similar. Hence, a decision based on visual inspection is unreliable. Also, given an arbitrary image pattern, no original to compare with being available, it should be possible to choose on a quantitative basis an appropriate model from a family of such models. In the context of this paper, different CMRF models should be interpreted as representing different neighbor sets N_1.

The problem of choosing appropriate CMRF models has received some attention in the literature. The possible approaches are using pairwise hypothesis testing procedures [7-8], Akaike's information criterion (AIC) [16], and the Bayes approach [11] [17]. The main criticisms of the pairwise hypothesis testing approach for CMRF model selection are that the resulting decision rules are not transitive [11], and the decision rules are not consistent, i.e., the probability of choosing an incorrect model does not go to zero even as the number of observations goes to infinity. When coding schemes are used, more than one coding scheme results for the same data. It might well turn out that the null hypothesis may be rejected on some coding schemes but accepted on some other coding schemes for the same data; some ad hoc methods leading to very conservative decisions [7-8] are used to overcome this problem. When it is required to choose between a first order CMRF model and a second order CMRF model, to ensure that the likelihoods are comparable, the coding scheme corresponding to the second order CMRF model should be used for the first order CMRF model also, causing further loss of efficiency in estimation for the first order model. Also, from a philosophical point of view, the pairwise hypothesis testing procedure is not suitable for choosing from a family of different CMRF models. This methodology is well suited for classical problems such as drug testing where the distribution under the null hypothesis that the drug is not effective is specified and the consequences of rejecting the null hypothesis when it is not true (type I error) are crucial. On the other hand, in the model selection problem, we are only interested in choosing a model that can reasonably account for the observed statistical phenomena. When the problem is posed in such a way that the null hypothesis is a specified CMRF model, and the alternative is its negation, the consequences of type I error are not serious. In other words, we cannot take a subjective view of any specific hypothesis, or equivalently any specific CMRF model, as is implicitly required in pairwise hypothesis testing procedures.

The model selection problem comes under the category of multiple decision problems. A method that is well suited for this problem would be to compute a test statistic for different models and choose the one corresponding to the minimum. The AIC criterion and the Bayes method are two such procedures. The AIC statistics can be formulated from the expression of the log likelihood function given in Section 2; the best model is the one which minimizes the AIC statistic. The method gives transitive decision rules but is not consistent even for one-dimensional autoregressive models [18]. Hence it is not advisable to use the AIC method for CMRF model selection.

We formulate the problem and suggest a decision statistic that is consistent, transitive and parsimonious. The actual derivation can be done by using standard Bayes decision theory, as was done for SAR models in [17]. Suppose we have three sets N_{11}, N_{12}, N_{13} of neighbors containing m_1, m_2, m_3 members, respectively. Corresponding to each N_{1q}, we write the CMRF model as

$$y(s) = \sum_{(k,\ell)\in N_{1q}} \theta_{q,k,\ell}\, [y(s + (k,\ell)) + y(s-(k,\ell))] + \sqrt{\nu_q}\, e(s),$$

$$\theta_{q,k,\ell} \neq 0,\ (k,\ell)\in N_{1q},\ ,\nu_q > 0,\ q = 1,2,3$$

Then the decision rule [4] for the choice of appropriate neighbors is: choose the neighbor set N_{1q^*} if

$$q^* = \arg\min_q \{C_q\} \tag{3.2}$$

where

$$C_q = -2\sum_{\Omega} \ell n(1-\hat{\underset{\sim}{\theta}}{}_q^T \underset{\sim}{\phi}_{qij}) + M^2 \ell n \hat{\nu}_q + 2m_q \ell n(M^2) \tag{3.3}$$

where

$$\hat{\underset{\sim}{\theta}}{}_q^T = \mathrm{Col}[\hat{\theta}_{q,k,\ell}\ ,\ (k,\ell)\in N_{1q}]$$

and

$$\underset{\sim}{\phi}_{q,i,j} = \mathrm{Col}[\cos \frac{2\pi}{M}((i-1)k + (j-1)\ell), (k,\ell)\in N_{1q}]$$

The model selection procedure consists of computing C_q for different models, and choosing the one corresponding to the lowest C_q. The factor 2 appears in the third term of (3.3) to account for the fact that in our notation m_q refers to the number of symmetric neighbors.

4. Experimental Results

We describe the results of some experiments regarding the generation of two-dimensional patterns obeying known CMRF models and estimation schemes developed in Section 2.

Experiment 1: Synthetic generation of two-dimensional Markov patterns.

From (2.7) we have

$$H(\underset{\sim}{\theta})\underset{\sim}{y} = \sqrt{\nu}\,\underset{\sim}{e} \tag{4.1}$$

where $\underset{\sim}{e}$ is a zero mean correlated noise sequence with correlation matrix $E(\underset{\sim}{e}\underset{\sim}{e}^T) = H(\underset{\sim}{\theta})$. The synthetic generation is then done by assigning some arbitrary values in the stationary region to $\underset{\sim}{\theta}$, and using the correlated noise vector $\underset{\sim}{e}$. The sequence $\{\underset{\sim}{e}(s)\}$ may be generated by using DFT computations [19], and $\underset{\sim}{y}$ can be generated by inverting the matrix $H(\underset{\sim}{\theta})$. Since $H(\underset{\sim}{\theta})$ is a block-circulant matrix, Fourier computations can be used for generating $\underset{\sim}{y}$. An alternative procedure developed in [4] is used in this paper. Before proceeding further, we define the following quantities: denote the M^2 Fourier vectors $\underset{\sim}{f}_{ij}$, $1 \leq i,j \leq M$ by

$$\underset{\sim}{f}_{ij} = \mathrm{col}[\underset{\sim}{t}_j, \lambda_i \underset{\sim}{t}_j, \ldots, \lambda_i^{M-1} \underset{\sim}{t}_j],$$

$$t_j = \mathrm{col}[1, \lambda_j, \lambda_j, \ldots, \lambda_j^{M-1}], \text{M-vector}$$

$$\lambda_i = \exp[\sqrt{-1}\ \frac{2\pi}{M}(i-1)]$$

The synthetic generation scheme then is as follows:

$$\underset{\sim}{y} = \sum_{\Omega}(\underset{\sim}{f}_{ij}x_{ij}/\sqrt{\mu'_{ij}(\underset{\sim}{\theta})}) + \alpha\underset{\sim}{1} \qquad (4.2)$$

where

$$x_{ij} = \frac{\sqrt{\nu}}{M^2}\sum_{\Omega}\underset{\sim}{f}_{ij}^{*T}\,\underset{\sim}{w}$$

$$\underset{\sim}{1} = (1,1,\ldots,1)\ ,\ M^2\text{-vector}$$

and $\mu'_{ij}(\underset{\sim}{\theta})$, $1\le i,j\le m$, are defined in (2.4) and $\underset{\sim}{w}$ is an identical and independently distributed noise sequence of known distribution. For a proof that such a $\underset{\sim}{y}$ indeed obeys underlying CMRF model, see [4].

We generate the vector $\underset{\sim}{w}$ using a Gaussian pseudo-random number generator, generate its Fourier sequence $\{x_{ij}\}$ by a two-dimensional FFT, and finally use (4.2). Sixteen such 64×64 images were generated using CMRF models with different neighbor sets and parameters. The gray scale values of the images were scaled to lie in the range 0-63. Alternatively, by multiplying the value of ν used by an appropriate constant the same patterns are obtained without scaling. The details of the models are given in Table I and the corresponding images are shown in Fig. 1. It can be seen that the patterns generated are quite varied and some of them look similar to real textures. Contrary to the existing belief [20] that spatial autoregressive models are incapable of exhibiting the local pattern replication attribute considered an essential ingredient of texture, some of the windows do exhibit repetitive patterns.

We use matrix notation in referring to windows of images. Diagonal neighbor sets seem to induce diagonal patterns, as in the windows in positions (1,2),(1,3),(2,2) and (2,3). The pattern in the window (2,1) corresponding to a CMRF model with $N_1 = \{(0,1),(1,0),(-1,1),(1,1),(0,2),(2,0)\}$ can also be generated by a SAR model as shown in [12] with $N = \{(0,1),(0,-1),(-1,0),(1,0)\}$ and $\theta_{-1,0} = \theta_{1,0} = -.12$, $\theta_{0,-1} = \theta_{0,1} = .28$ using the procedure outlined in [4-5]. Note that this pattern and the one in window (2,1) of Fig. 1 in [12] are very similar. Likewise the diagonal pattern (2,2) generated by a CMRF model with $N_1 = \{(-1,1),(1,1),(2,0),(0,2),(2,-2),(2,2)\}$ can also be generated by a SAR model with $N = \{(-1,1),(1,-1),(1,1),(-1,-1)\}$, $\theta_{-1,1} = \theta_{1,-1} = -.14$ and $\theta_{1,1} = \theta_{-1,-1} = .28$. However, the patterns in windows (1,1) and (4,4) corresponding to a CMRF model with $N_1 = \{(0,1),(1,0)\}$ cannot be generated by any finite parameter SAR model.

The role played by adding nearest neighbors can be illustrated using patterns (4,2),(4,3) and (2,1). Window (4,2) corresponds to $N_1 = \{(1,0),(0,1),(2,0),(0,2)\}$ and produces horizontally oriented, macrostructured strip patterns. By adding the symmetric neighbor (1,1) a diffused version in window (4,3) is produced. When an extra symmetric nearest neighbor (1,-1) is added we obtain a more microstructured pattern resembling water. By adding similar neighbors to the model in (4,1), vertically oriented patterns can be produced.

Experiment 2: The role of parameter values in the structure of patterns.

To illustrate the role played by the coefficients in generating the two-dimensional patterns, we consider the pattern corresponding to the CMRF model $N_1 = \{(-1,1),(1,1)\}$, $\alpha = 30.000$, $\nu = 1.1111$, $\theta_{-1,1} = .28$ and $\theta_{1,1} = -.14$. The values of the parameters tried are given in Table II and the resulting pictures in Fig. 2. Note that as the parameters are varied, the basic pattern is still retained but the "busyness" of the pattern is varied.

All the patterns considered thus far were generated using the same pseudorandom number generator. As illustrated in Figs. 4 and 6 of [12] for SAR models, different psuedorandom sequences produce very small perturbations in the patterns.

Experiment 3:

To test the usefulness of the estimation scheme and the choice of appropriate neighbor sets, experiments were done with one synthetic pattern. The true model used to generate the test pattern corresponds to $N_1 = \{(-1,1),(1,1)\}$, $\alpha = 30.00$, $\nu = 1.1111$, $\theta_{-1,1} = -.14$, $\theta_{1,1} = .28$. Using this CMRF model, the synthetic image (1,1) in Fig. 3 was generated. But for correct inference purposes regarding the estimation schemes, the original window (values not scaled for display purposes) was used. For estimation of the parameters, the sample mean of the window was subtracted and (2.14) and (2.15) were used. The test statistic C_q in (3.3) was also computed. The actual values of the estimates corresponding to different neighbor sets are given in Table III and the corresponding reconstructed images are shown in Fig. 3. Table III shows that the estimated values corresponding to the true neighbor sets are close to the true values. Note that when extra neighbors are added, the corresponding parameter values are very small. The decision statistics corresponding to the models considered are given in Table IV and the decision rule (3.2) picks up the true model.

Fig. 3 shows the images corresponding to the different neighbor sets in Table III using an identical array of noise variables. Windows (1,2) and (1,3) correspond to CMRF models that are subsets of the true model and are not as good as the original in (1,1). Window (1,4) is generated by the true neighbor set with estimated parameters and is very similar to (1,1). The pattern (2,1) corresponds to $N_1 = \{(1,0),(0,1)\}$, which has no common neighbors with the true neighbor set and is distinctly different from the original. The windows (2,2), (2,3) and (2,4) look close to the originals since the generating models include the original neighbor set. But the decision rule suggested here correctly eliminates these models.

Experiment 4: Unilateral approximations to CMRF models

Unilateral approximations to CMRF models with general neighbor sets of dependence are of interest for several reasons. First, the estimation of parameters in unilateral spatial autoregressive models can be handled similarly to one-dimensional autoregressive time series models. Secondly, unilateral neighbor sets make possible a state space representation of the images, which is useful in Kalman filtering of images [13]. Though it would be futile to expect that "good" unilateral approximations can be found for any arbitrary CMRF model, it would be worthwhile to explore the adequacy of such approximations even for some specific CMRF models. We consider the simplest CMRF model with $N_1 = \{(0,1),(1,0)\}$. As observed in [8] several unilateral models driven by white noise can be constructed in increasing order of complexity. The simplest unilateral model involves dependence on the north and west neighbors, the next approximation involves dependence on the north, west and southwest neighbors, and so on. In Fig. 4, we have shown patterns corresponding to several such approximate models. Window (1,1) corresponds to the original pattern, and (1,2) to the same neighbor set as in (1,1) but with estimated parameters. The pattern in (1,3) corresponds to the unilateral neighbor set $N = \{(-1,0),(0,-1)\}$; the parameters are estimated using the least squares method and the pattern was reconstructed using the method in [5]. This pattern looks surprisingly similar to (1,1) and (1,2). A careful analysis indicates that this is to be expected. This unilateral process is defined such that the probability distribution of $y(i,j)$ given all the observations in the unilateral region is equal to the probability distribution of $y(i,j)$ given $y(i-1,j), y(i,j-1)$. However, for the same unilateral neighbor set, the conditional distribution of $y(i,j)$ given all others except $y(i,j)$ depends [8,21] on $y(i-1,j), y(i+1,j), y(i,j-1), y(i,j+1), y(i-1,j+1)$, and $y(i+1,j-1)$, a CMRF model that "includes" the original CMRF model with $N_1 = \{(0,1),(1,0)\}$. As illustrated in Experiment 3, the patterns corresponding to a CMRF model with a neighbor set N_{11} and another CMRF model that includes N_{11} and some extra neighbors, look very similar. Hence, the simplest unilateral neighbor set $N = \{(-1,0),(0,-1)\}$ seems to be an excellent approximation to a first order CMRF model. Several other unilateral approximate patterns and bilateral patterns corresponding to $N = \{(0,1),(1,0),(0,-1),(-1,0)\}$, a SAR model, are shown in Fig. 4. The details of the models may be found in Table V. However, in more general situations, the procedure suggested in [1], based on approximating the spectral density of a CMRF model by successive approximations of unilateral neighbor sets, should be used. The goodness of approximation can be visually evaluated as described above.

By comparison, the unilateral approximations with $N = \{(0,-1),(-1,0)\}$ or $N = \{(0,-1),(-1,0),(-1,1)\}$ may not be good for SAR models with $N = \{(-1,0),(1,0),(0,1),(0,-1)\}$, since the conditional distribution of the latter model includes more neighbors than the conditional distribution corresponding to either of the former models.

5. Conclusions

We have considered some aspects of statistical inference methods applied to two-dimensional discrete Markov random fields. Specifically, we have considered estimation schemes for the parameters of the Markov model and decision rules for the choice of an appropriate model. We have illustrated the usefulness of the methods by using synthetic patterns. Currently, work is in progress on testing the estimation schemes using real textures.

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Table I. Details of CMRF models corresponding to Fig. 1.
For all the models α=30.00, ν=1.1111.

Image (Fig. 1)	Neighbor set N_1	Parameters
(1,1)	(1,0),(0,1)	$\theta_{1,0}$=.2794, $\theta_{0,1}$=.1825
(1,2)	(-1,1),(1,1)	$\theta_{-1,1}$=.28, $\theta_{1,1}$=-.14
(1,3)	(-1,1),(1,1)	$\theta_{-1,1}$=-.14, $\theta_{1,1}$=.28
(1,4)	(1,0),(1,-1),(0,1),(1,1)	$\theta_{1,0}$=.3357, $\theta_{1,-1}$=-.25 $\theta_{0,1}$=.3246, $\theta_{1,1}$=-.2126
(2,1)	(1,0),(1,-1),(0,1), (1,1),(2,0),(0,2)	$\theta_{1,0}$=-.2061, $\theta_{1,-1}$=.0536, $\theta_{0,1}$=.4467, $\theta_{1,1}$=.0536, $\theta_{2,0}$=-.0123, $\theta_{0,2}$=-.0580
(2,2)	(1,-1),(1,1),(2,0), (0,2),(2,-2),(2,2)	$\theta_{1,-1}$=-.2341, $\theta_{1,1}$=.4682 $\theta_{2,0}$=.0655, $\theta_{0,2}$=.0655 $\theta_{2,-2}$=-.0163, $\theta_{2,2}$=-.0655
(2,3)	(-1,1),(1,1)	$\theta_{-1,1}$=.28, $\theta_{1,1}$=-.22
(2,4)	(1,0),(0,1),(2,0),(0,2), (3,0),(0,3),(4,0),(0,4)	$\theta_{1,0}$=.12, $\theta_{0,1}$=-.10, $\theta_{2,0}$=.08, $\theta_{0,2}$=-.09, $\theta_{3,0}$=-.11, $\theta_{0,3}$=.11, $\theta_{4,0}$=-.07, $\theta_{0,4}$=.09
(3,1)	(1,0),(0,1),(3,0),(0,3)	$\theta_{1,0}$=.16, $\theta_{0,1}$=.10, $\theta_{3,0}$=.12, $\theta_{0,3}$=-.14
(3,2)	(1,0),(0,1),(3,0),(0,3)	$\theta_{1,0}$=.10, $\theta_{0,1}$=.16, $\theta_{3,0}$=-.14, $\theta_{0,3}$=.12
(3,3)	(1,0),(0,1),(1,1) (-1,1),(3,0),(0,3)	$\theta_{1,0}$=.12, $\theta_{0,1}$=-.10, $\theta_{1,1}$=.08, $\theta_{-1,1}$=-.09, $\theta_{3,0}$=-.11, $\theta_{0,3}$=.11

Image (Fig. 1)	Neighbor set N_1	Parameters
(3,4)	(0,2),(2,0)	$\theta_{0,2}=.2794$, $\theta_{2,0}=.1825$
(4,1)	(1,0),(0,1),(2,0),(0,2)	$\theta_{1,0}=.12$, $\theta_{0,1}=-.24$,
		$\theta_{2,0}=.16$, $\theta_{0,2}=-.18$
(4,2)	(1,0),(0,1),(2,0),(0,2)	$\theta_{1,0}=-.24$, $\theta_{0,1}=.12$,
		$\theta_{2,0}=-.18$, $\theta_{0,2}=.16$
(4,3)	(1,0),(0,1),(1,1),(2,0)	$\theta_{1,0}=-.24$, $\theta_{0,1}=.12$
	(0,2)	$\theta_{1,1}=.11$, $\theta_{2,0}=-.18$, $\theta_{0,2}=.16$
(4,4)	(0,1),(1,0)	$\theta_{0,1}=-.12$, $\theta_{1,0}=.18$

Table II. Values of parameters used to illustrate the role played by the coefficients in synthesizing patterns. $\alpha=30.00$, $\nu=1.1111$, $N_1=\{(-1,1),(1,1)\}$

Image (Fig. 2)	Parameters $\theta_{-1,1}$	$\theta_{1,1}$
(1,1)	.28	-.14
(1,2)	.24	-.14
(1,3)	.20	-.14
(1,4)	.16	-.14
(2,1)	.28	-.10
(2,2)	.28	-.06
(2,3)	.28	-.18
(2,4)	.28	-.22
(3,1)	.26	-.20
(3,2)	.24	-.18
(3,3)	.32	-.14
(3,4)	.34	-.16

Table III. *Details of models fitted to the synthetic* data generated by the model with $\theta_{-1,1}=-.14$, $\theta_{1,1}=.28$, $\alpha=30.0$, $\nu=1.1111$. The estimate $\hat{\alpha}$ is $=30.005$.

Image (Fig. 3)	Neighbor set N_1	$\hat{\nu}$	Estimates of Coefficients
(1,2)	(1,1)	1.1638	$\theta_{1,1}=.3116$
(1,3)	(-1,1)	1.3520	$\theta_{-1,1}=-.2093$
(1,4)	(-1,1),(1,1)	1.1033	$\theta_{-1,1}=-.1410$
			$\theta_{1,1}=.27875$
(2,1)	(0,1),(1,0)	1.4934	$\theta_{0,1}=-.0052$
			$\theta_{1,0}=-.0020$
(2,2)	(-1,1),(1,1),(1,0)	1.1033	$\theta_{-1,1}=-.14101$, $\theta_{1,1}=.27877$
			$\theta_{1,0}=-.0051$
(2,3)	(-1,1),(1,1),(0,1)	1.1033	$\theta_{-1,1}=-.1410$, $\theta_{1,1}=.27873$
			$\theta_{0,1}=-.001717$
(2,4)	(-1,1),(1,1),	1.1033	$\theta_{-1,1}=-.14101$,
	(1,0),(0,1)		$\theta_{1,1}=.27876$,
			$\theta_{1,0}=-.0049$,
			$\theta_{0,1}=-.0009$

Table IV. Test statistics corresponding to the fitted models in Table III.

Image (Fig. 3)	Test Statistic
(1,2)	1583.3
(1,3)	1637.2
(1,4)	1480.6
(2,1)	1676.3
(2,2)	1497.6
(2,3)	1497.1
(2,4)	1514.2

Table V. Unilateral approximations to the first order Markov model $\theta_{1,0}$=.2794, $\theta_{0,1}$=.1825, α=30.0, ν=1.1111. The estimate $\hat{\alpha}$ is 30.016.

Image (Fig. 4)	Neighbor set	$\hat{\nu}$	Estimates of Coefficients
(1,2)	(-1,0),(0,-1)	1.3117	$\theta_{-1,0}$=.3634, $\theta_{0,-1}$=.2550
(1,3)	(-1,0),(0,-1),(1,-1)	1.2991	$\theta_{-1,0}$=.3575, $\theta_{0,-1}$=.2166 $\theta_{1,-1}$=.0944
(1,4)	(-1,0),(-1,-1) (0,-1),(1,-1)	1.2991	$\theta_{-1,0}$=.3575, $\theta_{-1,-1}$=.0004, $\theta_{0,-1}$=.2165, $\theta_{1,-1}$=.0944
(2,1)	(-1,0),(-2,0),(-1,-1) (0,-1),(-2,-1),(1,-1)	1.2979	$\theta_{-1,0}$=.3460, $\theta_{-2,0}$=.0291, $\theta_{-1,1}$=-.0033, $\theta_{0,-1}$=.2159, $\theta_{-2,-1}$=.0025, $\theta_{1,-1}$=.0934
(2,2)	(-1,0),(-2,0),(-1,-1), (-2,-1),(0,-1),(0,-2), (1,-1),(1,-2)	1.2976	$\theta_{-1,0}$=.3457, $\theta_{-2,0}$=.0285, $\theta_{-1,-1}$=-.0040, $\theta_{-2,-1}$=.0019, $\theta_{0,-1}$=.2140, $\theta_{0,-2}$=.0030 $\theta_{1,-1}$=.0895, $\theta_{1,-2}$=.0127
(2,3)	(-1,0),(-2,0),(-1,-1), (-2,-1),(-1,-2),(-2,-2), (0,-1),(0,-2),(1,-1) (1,-2)	1.2974	$\theta_{-1,0}$=.3457, $\theta_{-2,0}$=.0287, $\theta_{-1,-1}$=-.0012, $\theta_{-2,-1}$=-.0038, $\theta_{-1,-2}$=-.0120 $\theta_{-2,-2}$=-.0028, $\theta_{0,-1}$=.2139, $\theta_{0,-2}$=.0076, $\theta_{1,-1}$=.0895, $\theta_{1,-2}$=.0132
(2,4)	(-1,0),(1,0), (0,1),(0,-1) (SAR model)	1.1582	$\theta_{-1,0}=\theta_{1,0}$=.1900, $\theta_{0,-1}=\theta_{0,1}$=.1387

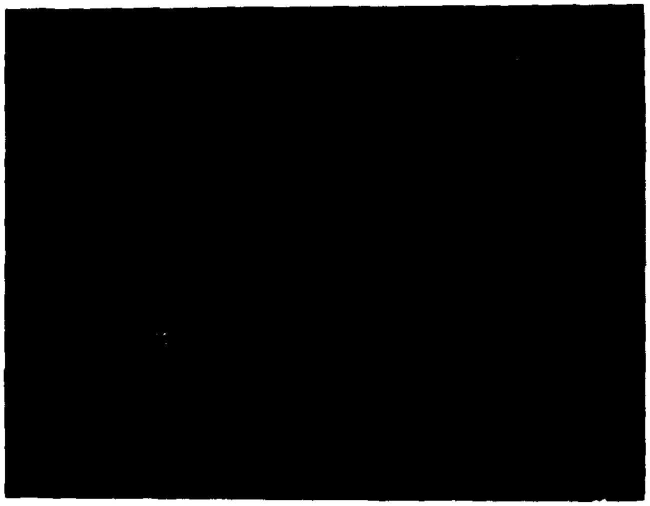

Figure 1. Examples of synthetic generation of patterns using CMRF models (for details of the models see Table I).

Figure 2. Patterns produced by models with the same neighbor set $N_1=\{(-1,1),(1,1)\}$, $\alpha=30.0$, $\rho=1.1111$ but with different sets of values for the coefficients (see Table II).

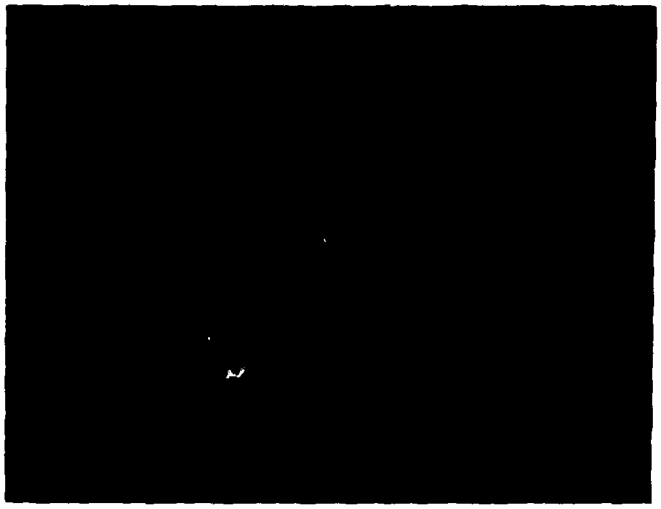

Figure 3. Synthetic patterns corresponding to original and different fitted CMRF models (see Table III for details of the fitted models).

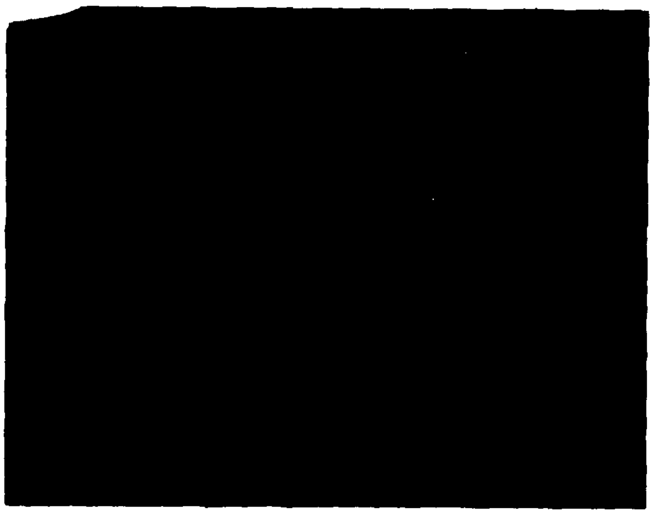

Figure 4. Several unilateral approximations to a first order CMRF model (see Table V for details of the unilateral models).

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We are interested in fitting two-dimensional, Gaussian conditional Markov random field (CMRF)models to images. The given finite image is assumed to be represented on a finite lattice of specific structure, obeying a CMRF model driven by correlated noise. The stochastic model is characterized by a set of unknown parameters. We describe two sets of experimental results. First, by assigning values to parameters in the stationary range, two-dimensional patterns are generated. It appears that quite a variety of patterns can be generated. Next,

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we consider the problem of estimating the unknown parameters of a given model for an image, and suggest a consistent estimation scheme. We also implement a decision rule to choose an appropriate CMRF model from a class of such competing models. The usefulness of the estimation scheme and the decision rule to choose an appropriate model is illustrated by application to synthetic patterns. Unilateral approximations to CMRF models are also discussed.

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